PMM U.S.S.R.,Vol.48,No.1,pp.18-22,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

SOME MATHEMATICAL PROBLEMS OF THE BATDORF-BUDIANSKY-MALMEISTER THEORY OF PLASTICITY*

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The Batdorf-Budiansky-Malmeister theory of plastic slip (the BBM theory) /1—3/, written in increment velocities in the Malmeister form, is considered. The properties of the corresponding equation of state

$$\varepsilon_{ij} = \alpha_{ijkh}\sigma_{kh}' + \left(\int\limits_{\omega} \frac{n_i\tau_j + n_j\tau_i}{2|\tau|} a(n,\sigma) \frac{n_k\tau_h}{|\tau|} dn\right)\sigma_{kh}' \tag{0.1}$$

are studied.

Here α_{ijkh} are the constants of elasticity, ε_{ij} is the strain rate tensor, σ_{kh} is the stress tensor, σ_{kh}' is its rate of change, $\tau_j = \tau_j(n_i, \sigma_{kh})$ is the tangential stress on an area with unit normal n_i, $a(n,\sigma)$ is a certain function, and ω is the domain on a unit sphere consisting of those normals n_i for which $|\tau| > c_*$ and $d|\tau|/dt > 0$.

Formula (0.1) yields a mapping that compares the tensor $\varepsilon_{ij}(t)$ with $\varepsilon_{ij}(0)$ and the tensor $\sigma_{ij}(t)$ differentiably dependent on the time variable t. The smoothness of this mapping and its fundamental properties are studied, and the correctness of the inverse problem of finding the tensor $\sigma_{ij}(t)$ by means of $\varepsilon_{ij}(t)$ and $\sigma_{ij}(0)$ is proved.

The methods developed in the paper are applicable to modifications of the BBM theory that differ from (0.1) (see /2/ apropos of these modifications).

1. The equation of state. The coefficients of elasticity in (0.1) are symmetric and elliptic, i.e.,

$$\alpha_{ijkh} = \alpha_{khij} = \alpha_{jikh} \tag{1.1}$$
$$\alpha_{ijkh}\sigma_{ij}\sigma_{kh} \geqslant \delta\sigma_{ij}\sigma_{ij}, \quad \delta > 0, \quad \forall\sigma \in l$$

where l is the space of symmetric (3 X 3)-matrices. The function $a(n,\sigma)$ is non-negative for $n \in \omega$ and has the follows form:

$$a(n,\sigma) = a_0(n, \sigma, |\tau(n,\sigma)|^2 - c_*^2), \; a_0(n,\sigma,0) \equiv 0 \tag{1.2}$$

where a_0 is a continuously differentiable function (see /1/, /2/, p.118, /3/). We use the notation

$$a_{ijkh}(\sigma,\sigma') = \int\limits_{\omega} \frac{n_i\tau_j + n_j\tau_i}{2|\tau|} a(n,\sigma) \frac{n_k\tau_h + n_h\tau_k}{2|\tau|} dn \tag{1.3}$$

where, as in (0.1), dn is the differential of the surface of the sphere

$$S^2 = \{n \in R^3 \,||\, n| = 1\}$$

We rewrite (0.1) as a differential equation in $\sigma_{kh}(t)$

$$\alpha_{ijkh}\sigma_{kh}' + a_{ijkh}(\sigma,\sigma')\sigma_{kh}' = \varepsilon_{ij} \tag{1.4}$$

we supplement it by the initial condition

$$\sigma_{ij}(0) = \sigma_{\bullet ij} \tag{1.5}$$

and we assume the tensor $\varepsilon_{ij}(t)$ to be continuous

$$\varepsilon_{ij}(t) \in C(0, t_0; l), \; t_0 > 0 \tag{1.6}$$

The main result of Sec.2, Theorem 5, asserts that (1.4) is solvable for σ_{ij}', i.e.,

$$\sigma_{ij}'(t) = B_{\sigma(t)}^{-1}(\varepsilon_{ij}(t)) \tag{1.7}$$

and the mapping

$$l \times l \to l, \; (\sigma, \mu) \to B_\sigma^{-1}(\mu)$$

satisfies the local Lipschitz condition. Because of the assumption (1.6) and the known results about the solvability of ordinary differential equations, we obtain that the solution of problem (1.7) and (1.5) exists locally and is unique.

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*Prikl.Matem.Mekhan.,48,1,38-43,1984

To prove that the solution exists in the whole segment $[0, t_0]$, we introduce a scalar product and norm in the space l

$$(\sigma, \tau)_l = \sigma_{ij}\tau_{ij}, \quad \| \sigma \|^2 = (\sigma, \sigma)_l$$

and we multiply (1.4) by $\sigma'_{ij}(t)$ scalarly in l

$$\alpha_{ijkh}\sigma'_{kh}\sigma_{ij}' + a_{ijkh}\sigma'_{kh}\sigma_{ij}' = \varepsilon_{ij}\sigma_{ij}' \tag{1.8}$$

Since the function $a(\sigma, n)$ is non-negative in the set ω, then

$$a_{ijkh}(\sigma, \sigma')\sigma'_{kh}\sigma_{ij}' = \int\limits_{\omega} a(\sigma, n)\left(\frac{n_k\tau_h}{|\tau|}\sigma'_{kh}\right)^2 dn \geqslant 0 \tag{1.9}$$

We obtain an *a priori* estimate of the solution from (1.1), (1.8) and (1.9)

$$\| \sigma(t) \| \leqslant \| \sigma(0) \| + \delta^{-1}\int\limits_0^t \| \varepsilon(\tau) \| d\tau$$

and the solution of problem (1.4), (1.5) exists globally.

Theorem 1. If condition (1.6) is satisfied, problem (1.4) and (1.5) has a unique continuously differentiable solution $\sigma_{ij}(t)$, $0 \leqslant t \leqslant t_0$.

Remark. Since (1.4) is equivalent to an ordinary differential equation with a non-linearity satisfying the local Lipschitz condition, the approximate solution of problem (1.4) and (1.5) can be found by the method of Euler broken lines. Here

$$\tau(0) = \sigma_0, \quad \tau\left(t_0\frac{r}{N}\right) = \tau\left(t_0\frac{r-1}{N}\right) + \frac{t_0}{N}\sigma', \quad 1 \leqslant r \leqslant N \tag{1.10}$$

where σ' is the solution of (1.4) for

$$\sigma = \tau\left(t_0\frac{r-1}{N}\right), \quad \varepsilon_{ij} = \varepsilon_{ij}\left(t_0\frac{r-1}{N}\right).$$

The sequence obtained as $N \to \infty$ approximates the value of the exact solution at the points $t_0 r/N$, $1 \leqslant r \leqslant N$. It follows from Theorem 4 (see below) that the σ' in (1.10) is the solution of the variational problem about minimizing a convex coercive functional in the space l

$$\frac{1}{2}\alpha_{ijkh}\sigma_{ij}'\sigma'_{kh} - \varepsilon_{ij}\left(t_0\frac{r-1}{N}\right)\sigma_{ij}' + \varphi\left(\sigma\left(t_0\frac{r-1}{N}\right), \sigma'\right) \to \inf, \quad r = 1, \ldots, N$$

Therefore finding the tensor $\sigma(t)$ numerically reduces to solving a number of variational problems.

2. Smoothness of the tensor $a_{ijkh}(\sigma, \sigma')$.

Let $\langle\cdot, \cdot\rangle$ and $|\cdot|$ denote the scalar product and norm in R^3, and $\sigma(n)$ the vector $\sigma_{ij}n_j$, where $\sigma \in l$ and $n \in R^3$. Then $\tau(n, \sigma) = \sigma(n) - \langle\sigma(n), n\rangle n$, and hence for $|n| = 1$ we have $|\tau(n, \sigma)|^2 = \langle\sigma(n), \sigma(n)\rangle - \langle\sigma(n), n\rangle^2$. This means

$$\omega = \omega_1 \cap \omega_2$$

$$\omega_1 = \{n \in S^2 \mid \psi_1(n, \sigma) > 0\}, \quad \psi_1(n, \sigma) = \langle\sigma^2(n), n\rangle - \langle\sigma(n), n\rangle^2 - c_*^2$$

$$\omega_2 = \{n \in S^2 \mid \psi_2(n, \sigma, \sigma') > 0\}$$

$$\psi_2(n, \sigma, \sigma') = \langle\sigma(n), \sigma'(n)\rangle - \langle\sigma(n), n\rangle \ \langle\sigma'(n), n\rangle$$

The boundary of the domain ω_1 is a smooth curve for those (σ, σ') for which the gradient of the function ψ_1 in $n \in S^2$ does not vanish on $\partial\omega_1$. The set m_1 of those (σ, σ') for which the system

$$\begin{gathered} |n|^2 = 1, \ \psi_1(n, \sigma) = 0 \\ \sigma^2(n) - 2\langle\sigma(n), n\rangle\, \sigma(n) - \varkappa n = 0 \end{gathered} \tag{2.1}$$

is solvable, is the complement to this set.

Since σ' is not in system (2.1), then $m_1 = m_{10} \times l$. Similarly, the bondary of the domain ω_2 is a smooth curve for $(\sigma, \sigma') \notin m_2$, where m_2 is the set of those (σ, σ') for which the following system is solvable

$$\begin{gathered} |n|^2 = 1, \ \psi_2(n, \sigma, \sigma') = 0 \\ (\sigma\sigma' + \sigma'\sigma)\, n - 2\langle\sigma(n), n\rangle\, \sigma'(n) - \\ 2\langle\sigma'(n), n\rangle\, \sigma(n) - \varkappa n = 0 \end{gathered} \tag{2.2}$$

Let $M_i \subset R^{16}$ be an algebraic set of solutions of system (2.1), $i = 1, 2$. Then m_i is the projection of M_i on $l \times l = R^{12}$ and according to the Zaidenberg-Tarski theorem /4/, will be a semi-algebraic set, i.e., a set of the form

$$\bigcup_{j=1}^{r} \{x \in R^{12} \mid p_{j1}(x) = \ldots = p_{ja(j)}(x) = 0;$$
$$q_{j1}(x) > 0, \ldots, q_{jb(j)}(x) > 0\}$$
$$r \geqslant 1, \; a(j),\; b(j) \geqslant 0;\; a(j) + b(j) \geqslant 1 \tag{2.3}$$

(p_{jk}, q_{jk} are polynomials). The set $M_i \subset R^{16}$ satisfies five equations. Hence, it is natural to except that $\dim M_i = 16-5 = 11$, and therefore, $\dim m_i \leqslant \dim M_i = 11$ (dim is dimensionality).

Lemma 1. The dimensionality of the semi-algebraic set m_i is not greater than 11; $i = 1,2$.

A detailed proof of the assertion of the lemma is awkward and fairly traditional. We present a sketch.

1) $i = 1$. We recall that $m_1 = m_{10} \times l$. By direct reasoning we see that m_{10} is contained in the set of symmetric 2-tensors for which two dertain eigenvalues agree. Consequently, $\dim m_{10} \leqslant 5$.

2) $i = 2$. We will eliminate x from system (2.2)

$$(\sigma\sigma^t + \sigma'\sigma)\, n + 2\langle\sigma^t(n), \;\sigma(n)\rangle\, n - 2\langle\sigma(n), \; n\rangle\, \sigma'(n) - 2\langle\sigma'(n), \; n\rangle\, \sigma(n) = 0$$
$$|n|^2 = 1 \tag{2.4}$$

Let $M(n)$ be the algebraic set of solutions of this system for fixed n. It is sufficient to show that $\dim M(n) \leqslant 9$. To do this, we select an orthonormal coordinate system in R^3 with unit direction e_1 equal to n. System (2.4) can be rewritten in the form

$$\gamma(\sigma)(\sigma'_{11}, \sigma_{12}', \sigma_{13}')^t = \eta(\sigma, \sigma_{ij}'), \quad i, j = 2, 3 \tag{2.5}$$

where $\gamma(\sigma)$ is a (3 x 3)-matrix and $\eta \in R^3$. It can be confirmed directly that the sets

$$\{\sigma \mid \mathrm{rk}\, \gamma(\sigma) = j\}, \quad j = 3, \; 2, \; 1, \; 0$$

have a dimensionality not higher than 6, 5, 4, and 0, respectively. For σ from these sets the dimensionality of the space of solutions $(\sigma_{11}', \sigma_{12}', \sigma_{13}')$ of (2.5) is not higher than 0, 1, 2, 3, respectively. Consequently, $M(n)$ is contained inthe union of surfaces of dimensionality not greater than 9, from which $\dim M(n) \leqslant 9$.

Let $m = m_1 \cup m_2$. It follows from the lemma proved there that m is a semi-algebraic manifold in whose representation in the form (2.3) there are no non-empty sets with $a(j) = 0$.

Lemma 2. The set m is contained in all 11-dimensional algebraic manifold m_0.

Proof. It is sufficient to set

$$m_0 = \bigcup_j \{x \mid p_{j1}(x) = \ldots = p_{ja(j)}(x) = 0\}$$

since $a(j) \geqslant 1$ for all j.

Corollary 1. The set m intersects almost every line in $l \times l$ parallel to the given line in not more than a finite number of points.

The assertion follows from the lemma and can be obtained by applying general facts about algebraic geometry. We will present a direct proof without using the concepts of algebraic geometry.

Let P be a vector in $l \times l$, π an orthogonal complement of P and $\Pi: l \times l \to \pi$, an operator orthogonal to the projection on π. Then the set of lines parallel to P is found to be in one-to-one correspondence with the elements of π. Let m_0 be the algebraic set from Lemma 2. It is known (/5/, Corollary (2.6) that $m_0 = m^1 \cup \cdots \cup m^s$, where $m^j, 1 \leqslant j \leqslant s$ are non-intersecting smooth manifolds. By virtue of Lemma 2 it can be assumed that $\dim m^j \leqslant 10$ for $j < s$, and either $\dim m^s = 11$, or $m^s = \phi$. Let $\mu_1 \subset \pi$ denote the set $\Pi(m^1 \cup \ldots \cup m^{s-1})$, and $\mu_2 \subset \pi$ the set of cirtical values for contraction of the mapping of Π on m^s (/6/,p.189). By virtue of the Sard lemma /6/, the set $\mu_1 \cup \mu_2 \subset \pi$ has a null 11-dimensional Lebesgue measure. If $x \in \pi \setminus (\mu_1 \cup \mu_2)$, then the line $\Pi^{-1}(x)$ does not intersect the sets $m^1, \ldots, m^{s-1}$, while the set m^s intersects not more than a countable number of points /6/. On the other hand, since $\Pi^{-1}(x) \cap m_0$ is an algebraic subset of the real line $\Pi^{-1}(x)$, it is either finite or equal to $\Pi^{-1}(x)$. Consequently, each line $\Pi^{-1}(x), x \in \pi \setminus (\mu_1 \cup \mu_2)$ intersects the set m_0 in not more than a finite number of points, and the corollary is proved.

Let $b_{ij}(\sigma, \sigma')$ denote the tensor

$$b_{ij}(\sigma, \sigma') \equiv a_{ijkh}(\sigma, \sigma')\, \sigma_{kh}' = \int_{\omega} \beta_{ij}(n, \sigma, \sigma')\, dn \tag{2.6}$$
$$\beta_{ij}'(n, \sigma, \sigma') = (n_i \tau_j + n_j \tau_i)\, a(n, \sigma, \psi_1)\, \psi_2 / (2\, |\tau|^2)$$

It follows from the form of the tensor β_{ij} and from condition (1.2) that

$$\beta_{ij}(n,\ \sigma,\ \sigma')=0,\ \forall n\in\partial\omega \tag{2.7}$$

Lemma 3. The tensor $b_{ij}(\sigma,\sigma')$ is continuous in $l\times l$.

Proof. If $(\sigma^{(1)},\sigma'^{(1)})$ is close to $(\sigma^{(2)},\sigma'^{(2)})$ then $\psi_q(n,\sigma^{(1)},\sigma'^{(1)})$ is close to $\psi_q(n,\sigma^{(2)},\sigma'^{(2)})$, $q=1,2$, while the tensor $\beta_{ij}(n,\sigma^{(1)},\sigma'^{(1)})$ is close to the tensor $\beta_{ij}(n,\sigma^{(2)},\sigma'^{(2)})$ uniformly in $n\in S^2$. Consequently, the contribution to $b_{ij}(\sigma^{(q)},\sigma'^{(q)})$, $q=1,2$ from the integrals over the domains $\omega(\sigma^{(1)},\sigma'^{(1)})\cap\omega(\sigma^{(2)},\sigma'^{(2)})$ are close.

We have

$$\psi_1(n,\ \sigma^{(\alpha)})>0,\quad \psi_1(n,\ \sigma^{(\beta)})\leqslant 0$$

in the set $\omega_1(\sigma^{(\alpha)})\setminus\omega_1(\sigma^{(\beta)})$, where $\alpha=1,\ \beta=2$, or $\alpha=2,\ \beta=1$.

Consequently, everywhere in $\omega_1(\sigma^{(\alpha)})\setminus\omega_1(\sigma^{(\beta)})$ the functions $\psi_1(\cdot,\sigma^{(q)})$ are uniformly small, and by virtue of (1.2) and (2.6), the tensors $\beta_{ij}(\cdot,\sigma^{(q)},\sigma'^{(q)})$, $q=1,2$ are small. Therefore, the contributions from the integrals over $\omega_1(\sigma^{(\alpha)})\setminus\omega_1(\sigma^{(\beta)})$ to $b_{ij}(\sigma^{(q)},\sigma'^{(q)})$ are small. Similarly, the contributions from the integrals over the domains $\omega_2(\sigma^{(\alpha)},\sigma'^{(\alpha)})\setminus\omega_2(\sigma^{(\beta)},\sigma'^{(\beta)})$ are small, from which the desired continuity follows.

Lemma 4. Everywhere in the domain $l\times l\setminus m_0$ the tensor $b_{ij}(\sigma,\sigma')$ has first derivatives bounded uniformly for $\|\sigma\|\leqslant C,\ \|\sigma'\|\leqslant C$. The following formulas hold:

$$\frac{\partial}{\partial\sigma_{pq}}b_{ij}(\sigma,\sigma')=\int_\omega\frac{\partial}{\partial\sigma_{pq}}\beta_{ij}(n,\sigma,\sigma')\,dn \tag{2.8}$$

$$\frac{\partial}{\partial\sigma'_{pq}}b_{ij}(\sigma,\sigma')=a_{ijpq}(\sigma,\sigma') \tag{2.9}$$

Proof. Let $(\sigma+\tau\sigma^{(1)},\sigma')\in l\times l\setminus m_0$ for $0\leqslant\tau\leqslant\tau_0$. Let $\omega(\tau)$ denote the set $\omega(\sigma+\tau\sigma^{(1)},\sigma')$. Then

$$b_{ij}(\sigma+\tau\sigma^{(1)},\sigma')-b_{ij}(\sigma,\sigma')=\int_{\omega(0)}\beta_{ij}(n,\sigma+\tau\sigma^{(1)},\sigma')-\beta_{ij}(n,\sigma,\sigma')\,dn+\int_{\Delta\omega}\beta_{ij}(n,\sigma+\tau\sigma^{(1)},\sigma')\,dn \tag{2.10}$$

where the second integral is taken over the algebraic difference in the sets $\Delta\omega=\omega(\tau)-\omega(0)$, i.e., the integral over $\omega(\tau)\setminus\omega(0)$ is taken with a plus sign, and over $\omega(0)\setminus\omega(\tau)$ with a minus sign.

The first integral in (2.10) equals

$$\tau\int_{\omega(0)}\left(\frac{\partial}{\partial\sigma_{pq}}\beta_{ij}(n,\sigma,\sigma'),\ \sigma^{(1)}_{pq}\right)_l+O(\tau^2) \tag{2.11}$$

To estimate the second integral, we note that $\nabla_n\psi_j(n,\sigma,\sigma')\neq 0$ for $n\in\partial\omega_j$. Consequently, for $0<\tau_0\ll 1$ the set $|\Delta\omega|$ is contained in a $c\tau$-neighbourhood of the boundary of the set $\omega(0)$ and $\operatorname{mes}|\Delta\omega|\leqslant c_1\tau$. On the other hand, by virtue of (2.7) the inequality $\|\beta_{ij}(n,\sigma+\tau\sigma^{(1)},\sigma')\|\leqslant c_2\tau$ holds in the $c\tau$-neighbourhood of the set $\partial\omega(\tau)$. Consequently, the second integral in (2.10) is $O(\tau^2)$, from which equality (2.8) also follows from (2.11). Equation (2.9) can be proved similarly.

Let the pairs of tensors $w_1,\ w_2\in l\times l$ be such that the line w_1+tw_2 intersects the set m at a finite number of points. Then the tensor $b_{ij}(w_1+tw_2)$ is continuous in t, and has everywhere except at a finite number of points a derivative uniformly bounded for uniformly bounded t. Therefore, this tensor equals the integral of its derivative with respect to t, and consequently is absolutely continuous. The set of points (w_1,w_2) with the necessary properties is compact in $(l\times l)^2$ because of Corollary 1. According to Lemma 4, the derivatives of the tensor b_{ij} are bounded in bounded sets. This means the tensor $b_{ij}(\sigma,\sigma')$ satisfies the local Lipschitz condition.

Theorem 2. The tensor $b_{ij}(\sigma,\sigma')$ satisfies a local Lipschitz condition in $l\times l$. Its first derivatives exist almost everywhere and are given by (2.8) and (2.9).

Let us determine the mapping $b_\sigma: l\to l,\ \mu\to b_{ij}(\sigma,\mu)$.

Theorem 3. The mapping b_σ is monotonic, i.e.,

$$(b_\sigma(\tau_1)-b_\sigma(\tau_2),\ \tau_1-\tau_2)_l\geqslant 0,\quad\forall\tau_1,\ \tau_2\in l$$

Proof. Let us fix $\nu\in l$. Then for almost all $(\mu,\sigma)\in l\times l$, the following computation holds:

$$(b_\sigma(\mu+\nu)-b_\sigma(\mu),\ \nu)_l=\left(\int_0^1\frac{d}{dt}b_\sigma(\mu+t\nu)\,dt,\ \nu\right)_l=\int_0^1 a_{ijkh}(\sigma,\mu+t\nu)\,\nu_{kh}\nu_{ij}\,dt\geqslant 0$$

in which (2.9) was used as well as the fact that the mapping $l \to l,\ \nu_{ij} \to a_{ijkh}(\sigma, \mu)\nu_{kh}$ is not a negative selfconjuage mapping. Now, the monotinicity of the mapping b_σ follows from its continuity.

Theorem 4. The function

$$\varphi(\sigma, \sigma') = \frac{1}{2}\int_\omega a(\sigma, n)\left(\frac{n_i\tau_j}{|\tau|}\sigma_{ij}'\right)^2 dn \tag{2.12}$$

is continuously differentiable with respect to σ' everywhere in $l \times l$ and is a potential for $b_{ij}(\sigma, \sigma')$, i.e.,

$$\frac{\partial}{\partial \sigma_{ij}'}\varphi(\sigma, \sigma') = b_{ij}(\sigma, \sigma') \tag{2.13}$$

Proof. Since the integral in (2.12) equals zero in the set $\partial\omega$, the reasoning from the proofs of Lemma 4 and Theorem 2 are applicable to (2.12). Hence, the function $\varphi(\sigma, \sigma')$ is continuous, absolutely continuous in almost all lines, and (2.13) holds almost everywhere. But the tensor $b_{ij}(\sigma, \sigma')$ is continuous, hence, (2.13) is satisfied everywhere and the function φ is continuously differentiable with respect to σ'.

Let α_{ijkh} be the coefficients of elasticity just as in (1.1). We obtain from Theorems 2 and 3 and the properties of monotonic mappings (/7/, Ch.3, Sec.2).

Corollary 2. For any $\sigma \in l$ the mapping

$$B_\sigma\colon l \to l,\ \ \mu_{ij} \to \alpha_{ijkh}\,\mu_{kh} + b_{ij}(\sigma, \mu)$$

satisfies the Lipschitz condition and has the reverse mapping B_σ^{-1}, which also satisfies the Lipschitz condition.

Let $\tau,\ \mu(1),\ \mu(2)$ and $\sigma(1),\ \sigma(2) \in l$ be such that $B_{\sigma(j)}(\mu(j)) = \tau,\ j = 1,2$. Then by virtue of (1.1).

$$\|\mu(j)\| \leqslant \sigma^{-1/2}\|\tau\|,\ i = 1,\ 2 \tag{2.14}$$

Moreover, multiplication of the equation

$$\alpha_{ijkh}(\mu(1) - \mu(2))_{kh} = -b(\sigma(1),\ \mu(1)) + b(\sigma(1), \mu(2)) - b(\sigma(1), \mu(2)) + b(\sigma(2), \mu(2))$$

by $\mu(1) - \mu(2)$ scalarly in l using inequalities (1.1), (2.14), the monotonicity of $b_{\sigma(1)}$ and Theorem 2, yields the estimate

$$\delta\|\mu(1) - \mu(2)\|^2 \leqslant (b(\sigma(2),\ \mu(2)) - b(\sigma(1),\ \mu(2)),$$
$$\mu(1) - \mu(2))_l \leqslant K\|\sigma(1) - \sigma(2)\|\,\|\mu(1) - \mu(2)\|$$

where the constant K depends on $\|\sigma(1)\|,\ \|\sigma(2)\|,\ \|\tau\|$. The assertion is obtained.

Theorem 5. The mapping $l \times l \to l,\ (\sigma, \mu) \to B_\sigma^{-1}(\mu)$ satisfies the Lipschitz condition in the variable μ and the local Lipschitz condition in the variable σ.

The author is grateful to R.V. Gol'dshtein who drew his attention to the BBM theory and assisted in numerous discussions, and also to A.N. Mokhel' for a number of useful comments.

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Translated by M.D.F.